

On the non-minimal character of the SMEFT

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When integrating out unknown new physics sectors, what is the minimal character of the Standard Model Effective Field Theory (SMEFT) that can result? In this paper we focus on a particular aspect of this question: “How can one obtain only one dimension six operator in the SMEFT from a consistent tree level matching onto an unknown new physics sector?” We show why this requires conditions on the ultraviolet field content that do not indicate a stand alone ultraviolet complete scenario. Further, we demonstrate how a dynamical origin of the ultraviolet scales assumed to exist in order to generate the masses of the heavy states integrated out generically induces more operators. Therefore, our analysis indicates that the infrared limit captured from a new sector in consistent matchings induces multiple operators in the SMEFT quite generically. Global data analyses in the SMEFT can and should accommodate this fact.

I. INTRODUCTION.

Despite the null results on beyond the Standard Model (SM) resonance searches from Run I and II at LHC, the arguments in favor of physics beyond the SM are very strong. It is reasonable to expect that the low energy effects of a new physics sector, which has a mass gap in its typical mass scale(s)¹ compared to the electroweak scale $v \simeq 246 \text{ GeV}$, could be resolved in the future. This is particularly the case if $\Lambda \lesssim 4\pi v$, which is consistent with expectations of ultraviolet (UV) physics motivated by naturalness concerns for the Higgs mass. Broad classes of new physics scenarios consistent with this minimal decoupling assumption can be constrained efficiently using effective field theory methods to analyse scattering data limited to energies $\sqrt{s} \sim v \ll \Lambda$. This formalism has come to be known as the Standard Model Effective Field Theory (SMEFT) recently [1–10] where the SM is supplemented with a series of higher dimensional operators²

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_5 + \mathcal{L}_6 + \mathcal{L}_7 + \mathcal{L}_8 + \dots \quad (1)$$

In this approach, the null results of lower energy tests for physics beyond the SM can be consistent with beyond-the-SM UV physics in the $\sim \text{TeV}$ mass scale range. However, a large set of experimental measurements that test the symmetry breaking patterns of the SM must be accommodated. This can be accomplished in a manner that avoids fine tuning. In this work, we use a symmetry assumption that new physics in the TeV mass range leads to a \mathcal{L}_6 correction to the SM that (approximately) respects the global symmetry group $G = \text{U}(1)_B \otimes \text{U}(1)_L \otimes \text{SU}(3)^5$, and in addition a discrete CP symmetry.³ Some of these symmetries cannot be exact, as the SM defines a minimal symmetry breaking in the SMEFT. However, the G and CP symmetry breaking pattern of the SM can allow the

effect of the UV physics sectors to be probed in the $\sim \text{TeV}$ scales of experimental interest because of two reasons: 1) it follows a Minimal Flavor Violating (MFV) pattern for flavor changing measurements [12–14]; and 2) it is proportional to the Jarlskog invariant [15, 16] in the case of SM CP violation.

In this paper, we consider matching patterns of operators that result from integrating out new physics sectors. We study the effect of simultaneously integrating out not only the heavy fields, but also a UV sector that generates the required heavy mass scale(s) $\Lambda > v$ under some generic considerations. We initially focus on the question of when, if ever, only one operator can be obtained in such a tree level matching in our chosen basis for \mathcal{L}_6 [2]. We examine this question in the SMEFT in terms of the matching effects of spin- $\{1, 1/2, 0\}$ fields that can couple to the SM through ($d \leq 4$) mass dimension interactions. Higher spin composite fields (and spin towers) are possible and even required in the presence of UV confining strong interactions. Similarly, when a UV sector with a strong interaction is present, there is no particular reason in general for tree level matchings, as opposed to non-perturbative matchings, to be the largest contribution to the Wilson coefficients. Such extra matching contributions are difficult to characterize (other than by using the full SMEFT formalism) and only reinforce our main point on the non-minimal character of the SMEFT, so we do not focus on these contributions.

The number of operators induced in matching is operator basis dependent. However, the conditions uncovered on the UV field content to reduce the operator profile are still meaningful. The conditions can be framed in terms of symmetries and several simple observations on new physics spectra and dynamics that can generate a scale Λ , as we will show. Practically speaking, most global analyses are being constructed using the well defined Warsaw basis [2], so we focus on this basis when examining the one operator question. We use the notation Q_i to denote an operator defined in the Warsaw basis in this work, and refer the reader to Ref. [2] for the explicit operator definitions. Note also that we refer to one operator with the understanding that, consistent with our assumptions of G symmetry, fla-

¹ Schematically denoted as Λ and assumed to be in the $\sim \text{TeV}$ range.

² The complete non-redundant sum of all operators at each mass dimension order d defines \mathcal{L}_d here.

³ This hypothesis is not the only way to accommodate TeV field content, see for example the discussion in Ref. [11].

vor indices are not used to distinguish operators.

The structure of this paper is as follows. Following a brief comment on the dimension-5 operator and Fermi theory in Section II, we provide in Section III.A a comprehensive discussion on the SMEFT matching at tree level onto \mathcal{L}_6 when a massive spin-1 state present in a UV physics sector is integrated out. We focus this discussion on the “one operator induced at tree level” question consistent with the assumed (approximate) G symmetry. We demonstrate why such a simple UV sector cannot be a complete scenario if a mechanism to generate the heavy state’s mass is demanded. We then discuss the spin-1/2 case, drawing a similar conclusions in Section III.B. In Section III.C we examine the case of integrating out a scalar field focused on the “one operator” question. We show how the scalar case is more subtle, but still argues for more operators when UV complete scenarios are demanded. Section IV contains our conclusions.

II. TWO EXCEPTIONAL EFT CASES

When considering the one operator question, we note that a few historical accidents in EFTs can be misleading. First of all, \mathcal{L}_5 and \mathcal{L}_d with $d \geq 6$ are distinct when considering this question. Due to the charges of the SM field content, only one operator (with flavor indices) can be constructed in \mathcal{L}_5 . The operator that results [17, 18],

$$\mathcal{L}_5 = \frac{c_{ij}}{2} \left(\bar{L}_i^c \tilde{H}^\star \right) \left(\tilde{H}^\dagger L_j \right) + \text{h.c.} \quad (2)$$

is the well known example where one operator at a particular mass dimension does result when integrating out UV physics.⁴ The interplay of global $U(1)_L$ number violation and the constraints of the SM field’s representations leading to one operator in \mathcal{L}_5 is an exception that is not repeated at higher orders in the SMEFT operator expansion [6–10].

Historically, Fermi theory has frequently been used as a prototypical EFT to build intuition. This can be unfortunate, as Fermi theory is atypical and has a number of non-trivial accidental features that are not generic. In Fermi theory, the four-fermion operator

$$Q_{\ell\ell} = \left(\bar{L}_\mu \gamma^\mu P_L L_\mu \right) \left(\bar{L}_e \gamma_\mu P_L L_e \right), \quad (3)$$

is generated when the W boson is integrated out. This effective operator is used in the process $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ to infer the Fermi constant, G_F . The UV sector in the case of Fermi theory is the SM which does induce a series of other operators at tree level, in addition to the operator $Q_{\ell\ell}$. These four-fermion operators are due to the Higgs field and the Z boson. However, the highly suppressed Yukawa couplings of the SM Higgs to light fermions leads to an

Case	SU(3) _C	SU(2) _L	U(1) _Y	G _Q	G _L	Couples to
$\mathcal{V}_I^{(1,8)}$	1,8	1	0	(1,1,1)	(1,1)	$\bar{d}_R \gamma^\mu d_R$
$\mathcal{V}_{II}^{(1,8)}$	1,8	1	0	(1,1,1)	(1,1)	$\bar{u}_R \gamma^\mu u_R$
$\mathcal{V}_{III}^{(1,8)}$	1,8	1	0	(1,1,1)	(1,1)	$\bar{Q}_L \gamma^\mu Q_L$
$\mathcal{V}_{IV}^{(1,8)}$	1,8	3	0	(1,1,1)	(1,1)	$\bar{Q}_L \sigma^I \gamma^\mu Q_L$
$\mathcal{V}_V^{(1,8)}$	1,8	1	0	(1,8,1)	(1,1)	$\bar{d}_R \gamma^\mu d_R$
$\mathcal{V}_{VI}^{(1,8)}$	1,8	1	0	(8,1,1)	(1,1)	$\bar{u}_R \gamma^\mu u_R$
$\mathcal{V}_{VII}^{(1,8)}$	1,8	1	-1	($\bar{3}$,3,1)	(1,1)	$\bar{d}_R \gamma^\mu u_R$
$\mathcal{V}_{VIII}^{(1,8)}$	1,8	1	0	(1,1,8)	(1,1)	$\bar{Q}_L \gamma^\mu Q_L$
$\mathcal{V}_{IX}^{(1,8)}$	1,8	3	0	(1,1,8)	(1,1)	$\bar{Q}_L \sigma^I \gamma^\mu Q_L$
$\mathcal{V}_X^{(3,6)}$	$\bar{3}$,6	2	-1/6	(1,3,3)	(1,1)	$\bar{d}_R \gamma^\mu Q_L^c$
$\mathcal{V}_{XI}^{(3,6)}$	$\bar{3}$,6	2	5/6	(3,1,3)	(1,1)	$\bar{u}_R \gamma^\mu Q_L^c$

TABLE I. Vector representations [21, 22] consistent with our assumptions. The first three rows are the same field sub-classified. Superscripts on the field label indicate the representation under color. The Gell-Mann matrix T^A (for both color and flavor 8’s) is present but suppressed in the coupling to some fermion bi-linears. σ_I is the Pauli matrix. The table largely follows from the SU(3) group relations $\mathbf{3} \otimes \mathbf{\bar{3}} = \mathbf{1} \oplus \mathbf{8}$ and $\mathbf{3} \otimes \mathbf{3} = \mathbf{6} \oplus \mathbf{\bar{3}}$.

exceptional situation numerically in terms of the operator profiles. The small Yukawa couplings are not formally the consequence of a fine tuning, as they are protected by the full chiral symmetry of the SM. More discussion on the accidents in Fermi theory, and how it is commonly misunderstood, can be found in Ref. [19].

Arguably, there is some theoretical evidence based on the structure and particle content of the SM in the direction of embedding this model into SU(5) or SU(10), see for example the arguments in Ref. [20]. This could be interpreted as a hint to an underlying theory, similar to the chiral structure of the SM being a low energy hint of its UV structure. However, the problems of TeV scale grand unified theories are very well known. In this work we make a more phenomenologically motivated choice and assume approximate G symmetry (and CP symmetry).

III. G SYMMETRIC TREE LEVEL MATCHINGS

A. Spin 1 states

Spin-1 fields that couple to the SM quark bi-linears in the manner assumed are given by Table I [21–23]. The requirement of linear couplings of mass dimension less than four, together with Lorentz symmetry and invariance under the full SM gauge group constrains the possible quantum numbers of UV field content. Fields with other representations that give SMEFT matchings respecting G are possible, if these conditions are relaxed. Our notation is that Q^c , L^c are the right handed conjugate doublet fields of the SM fermions. The global flavor symmetry in the

⁴ Here and below our notation with a c superscript indicates a charge conjugate representation of a SM field.

Case	SU(3) _C	SU(2) _L	U(1) _Y	G _Q	G _L	Couples to
$\mathcal{V}_I^{(1)}$	1	1	0	(1,1,1)	(1,1)	$\bar{e}_R \gamma^\mu e_R$
$\mathcal{V}_I^{(1)}$	1	1	0	(1,1,1)	(1,1)	$\bar{L}_L \gamma^\mu L_L$
$\mathcal{V}_{IV}^{(1)}$	1	3	0	(1,1,1)	(1,1)	$\bar{L}_L \sigma^I \gamma^\mu L_L$
\mathcal{V}_{XII}	1	2	3/2	(1,1,1)	($\bar{3}, \bar{3}$)	$\bar{L}_L^c \gamma^\mu e_R$
\mathcal{V}_{XIII}	1	1	0	(1,1,1)	(1,8)	$\bar{e}_R \gamma^\mu e_R$
\mathcal{V}_{XIV}	$\bar{3}$	2	-1/6	($\bar{3}, 1, 1$)	($\bar{3}, 1$)	$\bar{L}_L^c \gamma^\mu u_R$
\mathcal{V}_{XV}	$\bar{3}$	2	5/6	(1, $\bar{3}, 1$)	($\bar{3}, 1$)	$\bar{L}_L^c \gamma^\mu d_R$
\mathcal{V}_{XVI}	$\bar{3}$	1	-2/3	(1, $\bar{3}, 1$)	(1,3)	$\bar{e}_R \gamma^\mu d_R$
\mathcal{V}_{XVII}	$\bar{3}$	1	-5/3	($\bar{3}, 1, 1$)	(1,3)	$\bar{e}_R \gamma^\mu u_R$
\mathcal{V}_{XVIII}	3	2	-5/6	(1,1,3)	(1,3)	$\bar{e}_R \gamma^\mu Q_L^c$
\mathcal{V}_{XIX}	$\bar{3}$	1	-2/3	(1,1, $\bar{3}$)	(3,1)	$\bar{L}_L \gamma^\mu Q_L$
\mathcal{V}_{XX}	$\bar{3}$	3	-2/3	(1,1, $\bar{3}$)	(3,1)	$\bar{L}_L \sigma^I \gamma^\mu Q_L$

TABLE II. Different vector representations that couple to fermion bi-linears respecting G, without the insertion of a Yukawa matrix.

Case	SU(3) _C	SU(2) _L	U(1) _Y	G _Q	G _L	Couples to
$\mathcal{V}_I^{(1)}, \mathcal{V}_{II}^{(1)}, \mathcal{V}_{III}^{(1)}$	1	1	0	(1,1,1)	(1,1)	$H^\dagger i D^\mu H$
$\mathcal{V}_{IV}^{(1)}$	1	3	0	(1,1,1)	(1,1)	$H^\dagger \sigma^I i D^\mu H$
$\mathcal{V}_{XXI}^{(1)}$	1	1	-1	(1,1,1)	(1,1)	$H^T i D_\mu H$
$\mathcal{V}_{XXII}^{(1)}$	1	3	-1	(1,1,1)	(1,1)	$H^T i \sigma_I D_\mu H$

TABLE III. Vector representations coupling to currents constructed from Higgs fields.

quark and lepton sectors are defined as

$$G_Q = \text{SU}(3)_{\text{UR}} \times \text{SU}(3)_{\text{dR}} \times \text{SU}(3)_{\text{QL}}, \quad (4)$$

$$G_L = \text{SU}(3)_{\text{LL}} \times \text{SU}(3)_{\text{eR}}. \quad (5)$$

It is also possible to have a vector field couple to lepton bi-linears, to quark-lepton bi-linears or have an interaction with the SM Higgs field. We list the corresponding fields in Table II and Table III. Cases \mathcal{V}_{XII} , \mathcal{V}_{XIII} have fields that carry a global lepton number and $\mathcal{V}_{XIV} - \mathcal{V}_{XX}$ carry both lepton and baryon numbers. Although counterintuitive, UV fields that carry flavor quantum numbers do not necessarily lead to lower energy signatures of flavor violation – outside of the MFV pattern. Similarly, fields carrying lepton number do not necessarily lead to lower energy signatures of lepton flavor violation at tree level.⁵ Fields that carry both lepton and baryon number are potentially more problematic in inducing proton decay, but such phenomenological constraints are not the focus of this paper.

⁵ This was previously noted in Ref. [23] in the lepton number case for flavor singlet fields.

Case	Q_i generated at tree level
$\mathcal{V}_{IV}^{(1)}$	$Q_{ll}, Q_{qq}^{(1,3)}, Q_{lq}^{(1,3)}, Q_H, Q_{HD}, Q_{H\Box}, Q_{eH}, Q_{uH}, Q_{dH}$
$\mathcal{V}_{IV}^{(8)}$	$Q_{qq}^{(1)}, Q_{qq}^{(3)}$
$\mathcal{V}_{IX}^{(8)}$	$Q_{qq}^{(1)}, Q_{qq}^{(3)}$
\mathcal{V}_{XX}	$Q_{lq}^{(1)}, Q_{lq}^{(3)}$
$\mathcal{V}_{XXI}^{(1)}$	$Q_H, Q_{HD}, Q_{H\Box}, Q_{eH}, Q_{uH}, Q_{dH}$
$\mathcal{V}_{XXII}^{(1)}$	$Q_H, Q_{HD}, Q_{H\Box}, Q_{eH}, Q_{uH}, Q_{dH}$

TABLE IV. Examples of the sets of \mathcal{L}_6 operators in the SMEFT obtained by integrating out various massive vectors.

Dimension-6 operator matching

Solving the classical equations of motion (EOM) for the heavy vector fields and substituting the classical solution into the Lagrangian results in a direct tree level matching in terms of a product of currents. We define the currents as

$$J_a = \{J_\psi^\mu, J_H^\mu\} = \{\bar{\psi} \gamma_\mu \otimes \psi, (D_\mu H)^\dagger \otimes \Phi\}, \quad (6)$$

and the tree level matching is given by

$$\Delta \mathcal{L}_6 \supset -\frac{1}{M_V^2} (J_a^\mu)^\dagger J_b^\mu. \quad (7)$$

Here Φ represents H or $\tilde{H} = i \sigma_2 H^*$ and \otimes indicates a group product characterized by the SU(2)_L representation of vector fields.⁶ For a vector field of the form considered in Tables I, II and III, the current product falls into one of three types:

- four-fermion: $(J_\psi^\mu)^\dagger J_{\psi,\mu}$,
- scalar derivative: $(J_H^\mu)^\dagger J_{H,\mu}$,
- mixed scalar-fermion: $(J_\psi^\mu)^\dagger J_{H,\mu}, (J_H^\mu)^\dagger J_{\psi,\mu}$.

We have systematically examined the profile in terms of operators obtained in tree level matchings to the Warsaw basis from the fields listed in Tables I, II and III, finding the following rule:

Flavour singlet vector fields that do not break $G_Q \times G_L$ induce more than one operator at tree level when matching onto the SMEFT Warsaw basis.

This result is easy to demonstrate. Fields that are SU(3)_C and SU(2)_L singlets couple to (quark and lepton) fermion fields and also the scalar currents, inducing a large number of operators at tree level. A vector field can be made to couple to the left-handed doublets by assigning the field to a **3** of SU(2)_L. The scalar and leptonic couplings can be removed by assigning the field to a **8** of SU(3)_C. In

⁶ This notation is consistent with Ref. [23]. Note also that a further current of the form $D^\mu F_{\mu\nu}$ with $F = \{B, W, G\}$ is redundant [23].

Case	Op	U(1) _Y	G_Q, G_L Spurion
$\mathcal{V}_{\text{VIII}}^{(1)}$	$Q_{qq}^{(1)}$	0	$T^A Y_u^\dagger Y_u, T^A Y_d^\dagger Y_d$
$\mathcal{V}_{\text{IX}}^{(1)}$	$Q_{qq}^{(1)}$	0	$T^A Y_u^\dagger Y_u, T^A Y_d^\dagger Y_d$
\mathcal{V}_{XIX}	$Q_{lq}^{(1)}$	-2/3	/
$\mathcal{V}_{\text{X}}^{(3,6)}$	$Q_{qd}^{(1)}$	-1/6	/
$\mathcal{V}_{\text{XI}}^{(3,6)}$	$Q_{qu}^{(1)}$	5/6	/
$\mathcal{V}_{\text{XVIII}}$	Q_{qe}	-5/6	/
\mathcal{V}_{XII}	Q_{le}	3/2	/
\mathcal{V}_{XIV}	Q_{lu}	-1/6	/
\mathcal{V}_{XV}	Q_{ld}	5/6	/
$\mathcal{V}_{\text{V}}^{(1)}$	Q_{dd}	0	$T^A Y_d^\dagger Y_d$
$\mathcal{V}_{\text{VI}}^{(1)}$	Q_{uu}	0	$T^A Y_u^\dagger Y_u$
$\mathcal{V}_{\text{VII}}^{(1)}$	$Q_{ud}^{(1)}$	-1	$Y_d^\dagger Y_u$
$\mathcal{V}_{\text{XIII}}$	Q_{ee}	0	$T^A Y_e^\dagger Y_e$
\mathcal{V}_{XVI}	Q_{ed}	-2/3	/
$\mathcal{V}_{\text{XVII}}$	Q_{eu}	-5/3	/

TABLE V. Operators induced at tree level when the massive vector case is integrated out. The cases are grouped in the table into the chiral $(J_\psi^\mu)^\dagger J_{\psi,\mu}$ operator classes induced. The top section refers to LLLL operators. The middle section of the table refers to LLRR operators. The bottom section of the table refers to RRRR operators induced at tree level.

this case the operator profile is reduced to at least two $(J_\psi^\mu)^\dagger J_{\psi,\mu}$ operators via the relation [2]

$$(\bar{Q}_L^p \sigma^I T^A \gamma^\mu Q_L^r)(\bar{Q}_L^s \sigma^I T^A \gamma^\mu Q_L^t) = -\frac{1}{4} Q_{pqsr}^{(3)} + \frac{3}{4} Q_{pqsr}^{(1)} - \frac{1}{6} Q_{prst}^{(3)}, \quad (8)$$

here p, r, s, t are flavor indices. We show some examples of the multiple operators induced when integrating out vector fields at tree level in Table IV. Introducing $G_Q \times G_L$ symmetry, vector fields can be reduced in their infrared (IR) SMEFT operator profile to one operator in the Warsaw basis in the limit of vanishing Yukawa matrices; see Table V. Note that with the exception of case $\mathcal{V}_{\text{VII}}^{(1)}$ which has a bi-linear flavor breaking spurion in Y_d^\dagger and Y_u , the presence of a $U(1)_Y$ charge is also associated with the lack of Higgs scalar currents induced. This has an important consequence when the self interactions of the vector are studied for unitarity violation, as will be discussed shortly.

A spurion analysis allows the corrections due to the nonzero Yukawa matrices of the SM (that break the flavor symmetry in a phenomenologically safe MFV pattern) to be systematically studied. We define the SM Yukawa matrices Y_u, Y_d, Y_e as

$$\mathcal{L}_Y = -(Y_u)_r^p \bar{u}_{R,p} Q_L^r \tilde{H}^\dagger - (Y_d)_r^p \bar{d}_{R,p} Q_L^r H^\dagger - (Y_e)_r^p \bar{e}_{R,p} L_L^r H^\dagger + \text{h.c.} \quad (9)$$

$G_Q \times G_L$ symmetry is restored if we endow the Yukawa matrices with the transformation properties under $\{G_Q, G_L\}$

$$Y_u \sim (3, 1, \bar{3}, 1, 1), \quad Y_d \sim (1, 3, \bar{3}, 1, 1), \\ Y_e \sim (1, 1, 1, \bar{3}, 3). \quad (10)$$

Introducing $G_Q \times G_L$ symmetry breaking when the Y_u, Y_d, Y_e matrices take on their SM values gives more operators at tree level for fields with flavor quantum numbers. On general grounds, the $(J_H^\mu)^\dagger J_{H,\mu}$ current products are induced proportional to two spurions breaking the flavor symmetry, and the $(J_\psi^\mu)^\dagger J_{H,\mu}, (J_H^\mu)^\dagger J_{\psi,\mu}$ current products are induced proportional to one flavor breaking spurion insertion. Here we refer to the spurions listed in Table V that are bi-linear in Yukawa matrices. As a specific example consider $\mathcal{V}_{\text{VIII}}^{(1)}$ that is a **8** under $SU(3)_{Q_L}$. The Lagrangian⁷ is given by $\mathcal{L}_{\text{SM}} + \mathcal{L}_{\mathcal{V}_{\text{VIII}}^{(1)}}$ where

$$\mathcal{L}_{\mathcal{V}_{\text{VIII}}^{(1)}} = -\frac{1}{2} (D_\mu \mathcal{V}_\nu D^\mu \mathcal{V}^\nu - D_\mu \mathcal{V}_\nu D^\nu \mathcal{V}^\mu) - \frac{M_V^2}{2} \mathcal{V}_\nu \mathcal{V}^\nu \\ + (\lambda_V \mathcal{V}_{\mu,A} T^A Y_u^\dagger Y_u (D^\mu H)^\dagger H + \text{h.c.}), \quad (11) \\ + g_V \mathcal{V}_{\mu,A} (\bar{Q}_L T^A \gamma^\mu Q_L).$$

Note that the largest spurion that restores the flavor symmetry for the second line is $T^A Y_u^\dagger Y_u$ and some indices are suppressed in Eqn. (11). The additional spurion breaking proportional to $Y_d^\dagger Y_d$ is neglected in what follows. Integrating by parts and the on-shell condition for the vector field are used to manipulate the derivative to appear as shown on the second line in Eqn. (11). Integrating out the field $\mathcal{V}_{\text{VIII}}^{(1)}$ using the classical EOM gives

$$\Delta \mathcal{L}_6 \supset \frac{g_V^2}{4 M_V^2} \left[Q_{rs\bar{s}r}^{(1)} - \frac{1}{3} Q_{rr\bar{s}s}^{(1)} \right] \\ + \frac{1}{4 M_V^2} \left[((\text{Im} \lambda_V)^2 - (\text{Re} \lambda_V)^2) Q_{H\Box} + 4(\text{Im} \lambda_V)^2 Q_{HD} \right. \\ \left. + 2i(\text{Re} \lambda_V)(\text{Im} \lambda_V)(Y_b^\dagger Q_{bH} - Y_b Q_{bH}^\dagger) \right. \\ \left. - 2i(\text{Re} \lambda_V)(\text{Im} \lambda_V)(Y_u^\dagger Q_{uH} - Y_u Q_{uH}^\dagger) \right] \\ \times \left[\text{Tr}[(Y_u^\dagger Y_u)(Y_u^\dagger Y_u)] - \frac{(\text{diag}(Y_u^\dagger Y_u))^2}{3} \right] \quad (12) \\ - \frac{g_V \text{Im}[\lambda_V]}{2 M_V^2} Q_{Hq}^{(1)} \left[(Y_u^\dagger Y_u)_r^p - \frac{\text{diag}(Y_u^\dagger Y_u)}{3} \delta_{pr} \right] \\ + i \frac{g_V \text{Re}[\lambda_V]}{2 M_V^2} \left[((Y_u^\dagger Y_u) Y_a^\dagger)_i^m Q_{im}^{aH} - (Y_a (Y_u^\dagger Y_u))_m^i Q_{mi}^{aH} \right] \\ - i \frac{g_V \text{Re}[\lambda_V]}{6 M_V^2} \text{Tr}[Y_u^\dagger Y_u] \left[(Y_a^\dagger)_i^m Q_{im}^{aH} - (Y_a)_m^i Q_{mi}^{aH} \right],$$

where the dummy labels a and b are summed over $\{u, d\}$ and $\{e, d\}$, respectively. A similar pattern of matchings onto the class 3 ($D^2 H^4$), 5 ($H^3 \bar{\psi} \psi$) and 7 ($H^2 D \bar{\psi} \psi$) operators of the Warsaw basis is present for almost all color singlet fields with flavor quantum numbers listed in Tables II and III. The exceptional case is the field \mathcal{V}_{XII} whose non-trivial $SU(2)_L$ representation and $U(1)_Y$ charge forbids a scalar current from being induced at tree level in this manner.

The pattern of tree level matchings is strongly dictated by the charges and representations of the UV fields under

⁷ Recall the flavor adjoint **8** representation is real.

$SU(3)_C \times SU(2)_L \times U(1)_Y$, G_Q and G_L . We emphasize, data fits to subsets of operators in the SMEFT formalism can be justified by appealing to UV field content with $U(1)_Y$ charges and non-trivial representations under SM groups when only retaining tree level matching contributions. See Table V for details on cases that generate only one operator at a time.

This conclusion is subject to the following qualifications. First, the single operators obtained in tree level matchings to the vectors in Tables II, III are limited to $(J_\psi^\mu)^\dagger J_{\psi,\mu}$ operator forms. Such operators at LHC contribute to continuum parton production in a fashion dictated by the power counting of the theory. Conversely, the precise measurements made on a scattering through a SM resonance (with mass M and width Γ) parametrically has a Γ/M suppression, compared to the leading resonant behavior, when considering the interference with $(J_\psi^\mu)^\dagger J_{\psi,\mu}$ operators.

Second, as $y_t \simeq 1$, a flavor symmetry spurion breaking proportional to only powers of Y_u can induce operators of class 3, 5 and 7 without significant numerical suppression. This makes it difficult to justify “one at a time” data fits to $(J_\psi^\mu)^\dagger J_{\psi,\mu}$ SMEFT operators with up quark field content (consistent with our assumptions). On the other hand, one at a time data fits to $(J_\psi^\mu)^\dagger J_{\psi,\mu}$ operators that only have leptonic or down quark field content can be potentially justified. In these cases the induced scalar currents proportional to MFV like flavor breaking spurions are numerically suppressed compared to pure up quark spurions by at least $y_b/y_t \sim 10^{-2}$.

Finally, we also note that we never obtain only one operator in such a tree level matching that involves the Higgs field, in the cases of massive vector UV field content considered.

Arguments against orphaned vectors.

The vector fields listed in Tables I, II and III inducing a single \mathcal{L}_6 SMEFT operator at tree level, carry at least one non-trivial representation under the SM gauge symmetry and flavor symmetries.⁸ Non-trivial representations and $U(1)_Y$ charges reduce the interactions for SM particles with new sector, which consequently minimizes the IR SMEFT operator profile. However, such fields in general do not indicate a stand alone UV complete scenario (where the vector could be an “orphan”) for the following reasons.

(1) Landau poles and triviality. The β function of the coupling of the vector fields to the fermion bi-linears (denoted g_V in Eqn. (11)) is determined by renormalizing the fermion fields and vector field two point functions, and subsequently extracting the β function for g_V . We relate

⁸ In all cases but one, multiple non-trivial representations are present. The one exceptional case is \mathcal{V}_{XIII} which is only an **8** under $SU(3)_{eR}$.

Case	$\frac{16\pi^2 \epsilon \delta_{Z_3}}{g_V^2 \langle O \rangle}$	$\frac{-16\pi^2 \epsilon \delta_{Z_\psi}}{g_V^2}$	$\frac{-16\pi^2 \epsilon \delta_{Z_\psi}}{g_V^2}$	$\frac{-16\pi^2 \epsilon \delta_{Z_V}}{g_V^2}$	β_y
$\mathcal{V}_{VIII}^{(1)}$	$\mathcal{F}(3_{F1})$	$C_F^{(3_{F1})}$	$C_F^{(3_{F1})}$	$\frac{4}{3} \left(\frac{1}{2}\right)_{F1} \cdot 3_C$	+
$\mathcal{V}_{IX}^{(1)}$	$\mathcal{F}(2)\mathcal{F}(3_{F1})$	$C_F^{(2)} C_F^{(3_{F1})}$	$C_F^{(2)} C_F^{(3_{F1})}$	$\frac{2 \cdot 3_C}{3} \left(\frac{1}{2}\right)_{F1} \left(\frac{1}{2}\right)_L$	+
\mathcal{V}_{XIX}	1	$3_{F1} \cdot 3_C$	3_{F1}	$\frac{2}{3} \cdot 2$	+
$\mathcal{V}_{X,XI}^{(3)}$	-1_C	$3_{F1}(-2)_C \cdot 2$	$3_{F1}(-2)_C$	$\frac{2}{3} \cdot (-1)_C$	-
$\mathcal{V}_{X,XI}^{(6)}$	1	$3_{F1} \cdot 1_C \cdot 2$	$3_{F1} \cdot 1_C$	$\frac{2}{3} \cdot 1_C$	+
\mathcal{V}_{XVIII}	1	$3_{F1} \cdot 3_C \cdot 2$	3_{F1}	$\frac{2}{3}$	+
\mathcal{V}_{XII}	1	3_{F1}	$3_{F1} \cdot 2$	$\frac{2}{3}$	+
\mathcal{V}_{XIV}	1	$3_{F1} \cdot 3_C$	$3_{F1} \cdot 2$	$\frac{2}{3}$	+
\mathcal{V}_{XV}	1	$3_{F1} \cdot 3_C$	$3_{F1} \cdot 2$	$\frac{2}{3}$	+
$\mathcal{V}_V^{(1)}$	$\mathcal{F}(3_{F1})$	$C_F^{(3_{F1})}$	$C_F^{(3_{F1})}$	$\frac{2}{3} \left(\frac{1}{2}\right)_{F1} \cdot 3_C$	+
$\mathcal{V}_{VI}^{(1)}$	$\mathcal{F}(3_{F1})$	$C_F^{(3_{F1})}$	$C_F^{(3_{F1})}$	$\frac{2}{3} \left(\frac{1}{2}\right)_{F1} \cdot 3_C$	+
$\mathcal{V}_{VII}^{(1)}$	1	3_{F1}	3_{F1}	$\frac{2}{3} \cdot 3_C$	+
\mathcal{V}_{XIII}	$\mathcal{F}(3_{F1})$	$C_F^{(3_{F1})}$	$C_F^{(3_{F1})}$	$\frac{2}{3} \left(\frac{1}{2}\right)_{F1}$	+
\mathcal{V}_{XVI}	1	$3_{F1} \cdot 3_C$	3_{F1}	$\frac{2}{3}$	+
\mathcal{V}_{XVII}	1	$3_{F1} \cdot 3_C$	3_{F1}	$\frac{2}{3}$	+

TABLE VI. One loop renormalization results. Here $\langle O \rangle$ indicates the matrix element of the vector-fermion bilinear interaction term and δ_{Z_3} corresponds to the divergence present in this three point interaction at one loop from the vector-fermion coupling. The notation is such that $\mathcal{F}(N) \equiv C_F^{(N)} - \frac{1}{2}N$ with $C_F^{(N)} = \frac{N^2-1}{2N}$. We have labeled several of the numerical factors in the table with the group space ($SU(3)_C$, $SU(3)_{F1}$, $SU(2)_L$) that generates them, with the subscript *F1* indicating a $SU(3)$ flavour group.

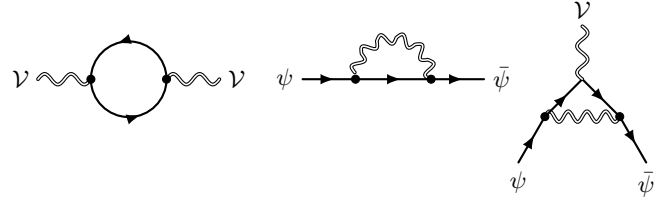


FIG. 1. Diagrams relevant for the renormalization of g_V .

the bare (0) and renormalized (r) fields and couplings as

$$V_\mu^{(0)} = \sqrt{Z_V} V_\mu^{(r)}, \quad g_V^{(0)} = Z_{g_V} g_V^{(r)} \mu^\epsilon, \quad (13)$$

$$\psi_i^{(0)} = \sqrt{Z_{\psi_i}} \psi_i^{(r)}, \quad (14)$$

where $Z_x = 1 + \delta_{Z_x}$ for $x = \{V, g_V, \bar{\psi}, \psi\}$. We use a renormalization scheme employing \overline{MS} subtraction and $d = 4 - 2\epsilon$ dimensions. The factor of μ^ϵ is introduced in g_V to render the renormalized coupling dimensionless. We use the standard result extracting the β function for composite operators due to ϵ poles in dimensional regularization [24]. The relevant diagrams are shown in Figure 1. The β -function for the running of the coupling g_V is given by

$$\beta_{g_V} = 2 g_V \epsilon \left(-\frac{\delta_{Z_3}}{\langle O \rangle} - \frac{1}{2} \delta_{Z_\psi} - \frac{1}{2} \delta_{Z_\psi} - \frac{1}{2} \delta_{Z_V} \right), \quad (15)$$

where the renormalization factors δ_Z 's for the various vector field cases are presented in Table VI. The general ex-

pectation is that g_V will have a positive β function – indicating Landau poles [25], quantum triviality [26] and a UV incompleteness. This is indeed the case for all vector fields inducing one $(J_\psi^\mu)^\dagger J_{\psi,\mu}$ operator, with the exception of color $\mathbf{\bar{3}}$ vectors coupling to quark bi-linears; i.e. cases $\mathcal{V}_{X,XI}^{\mathbf{\bar{3}}}$. In this exceptional case, the $SU(3)_C$ vector-fermion coupling mimics the effect of a non-abelian interaction.

An oversimplified UV scenario afflicted with an internal inconsistency indicated by the presence of Landau poles cannot formally generate a consistent IR limit. This indicates that further new physics must be present below the Landau pole scale Λ_L approximated by

$$\Lambda_L \sim M_V \exp [g_V / \beta_{g_V}]. \quad (16)$$

Although Landau poles clearly indicate an internal inconsistency within a UV scenario, numerically corrections suppressed by Λ_L are smaller than one loop matching effects.

(2) Unitarity and vector self-interactions. As in QCD, the running of the coupling g_V for the vector-fermion interactions can (at times) be made asymptotically free by introducing the self interactions of the vector field, with a global symmetry relating the vector self-couplings to g_V . However, a more intractable problem is generated by $\mathcal{O}(1)$ self interactions of orphan vector fields. The four point vector self interaction is not forbidden by any symmetry. Conversely the three point interaction can be forbidden by the presence of a $U(1)_Y$ charge in the composite field. Consider the $2 \rightarrow 2$ longitudinal vector scattering displayed in Fig. 2 that is dictated by such a four-point and three-point interactions at tree level. The relevant Lagrangian involving the vector self-interactions are

$$\Delta \mathcal{L}_V = \frac{\lambda}{4} \mathcal{V}_\mu^\dagger \mathcal{V}_\mu \mathcal{V}_\nu^\dagger \mathcal{V}_\nu + g' \partial_\mu \mathcal{V}^\mu \mathcal{V}_\nu^\dagger \mathcal{V}^\nu + \dots \quad (17)$$

The amplitudes at the leading order with the high-energy approximation for the vector polarization $\epsilon_L^\mu \simeq p^\mu / M_V$ through a s-, t- and u- channel vector exchange and a four-point contact interaction, respectively, read

$$\mathcal{M}_{3,s}^L = (g')^2 F_s \frac{st - su}{4 M_V^4}, \quad (18)$$

$$\mathcal{M}_{3,t}^L = (g')^2 F_t \frac{st - ut}{4 M_V^4}, \quad (19)$$

$$\mathcal{M}_{3,u}^L = (g')^2 F_u \frac{us - ut}{4 M_V^4}, \quad (20)$$

$$\mathcal{M}_4^L = \lambda \left(F_s \frac{t^2 - u^2}{4 M_V^4} + F_t \frac{s^2 - u^2}{4 M_V^4} + F_u \frac{s^2 - t^2}{4 M_V^4} \right). \quad (21)$$

Here abstract group structure constants $F_{s,t,u}$ for three channels have been introduced. For example, in the model $\mathcal{V}_{IX}^{(1)}$: $F_s = f^{ABE} f^{CDE} f^{ijn} f^{kln}$ where A, B, C, D, E refer to the flavor index and i, j, k, l, n denote the iso-spin index.

If $\lambda = (g')^2$ is accomplished by a global symmetry then the amplitudes \mathcal{M}_3^L will cancel with three terms in \mathcal{M}_4^L with an identical F factor respectively, through the Mandelstam relation $s + t + u = 4 M_V^2$. As a result, the leading

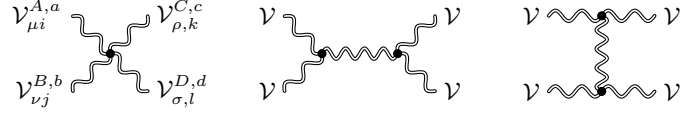


FIG. 2. $2 \rightarrow 2$ vector scattering diagrams.

scaling in $\sim s^2 / M_V^4$ disappears. The full amplitudes then grow as $\sim s / M_V^2$. However, if the three-point interaction is forbidden – for example due to the field carrying a $U(1)_Y$ charge – then the amplitude cannot be so moderated in its growth at high energies, and scales as $\sim (p^2)^2 / M_V^4$. In this manner, the presence of a $U(1)_Y$ charge forbidding the scalar current simultaneously turns off the three-point interaction that is required to moderate the high energy scattering behavior of an orphaned vector field.

Standard partial wave unitarity arguments [27–29] give that the unitarity violation scale associated with the vector field without a three point interaction is

$$\Lambda \lesssim 0.2 M_V (F_t + F_u)^{-1/4} \lambda^{-1/4}, \quad (22)$$

where F_t , an F_u are determined by a particular scattering cross section with fixed SM gauge and flavor indices that the vector carries. A quick onset of unitarity violation follows from the expectation of a sizable four-point interaction that is expected to emerge from a strongly interacting composite sector on general grounds. Even introducing a loop suppression to the vector self interaction, that is $\lambda \sim (16\pi)^{-1}$, is of little help – one still finds $\Lambda \sim M_V$ due to the presence of a fourth root in Eqn. (22). Hence, the UV strong sector should be simultaneously considered when integrating out to define a consistent IR matching onto the SMEFT. This would increase the low energy operator profile of such a scenario in the SMEFT beyond one operator generically due to non perturbative matchings, and a “one at a time” analysis invoking a tree level matching would be logically incoherent.

(3) Siblings of massive vectors with non-trivial representations. A massive vector field with non-trivial representations under subgroups of G is also generically accompanied by more “sibling” fields. If the massive vector gains a mass by a UV Higgs mechanism, the corresponding sibling field includes at least a scalar (S) obtaining a vacuum expectation value (vev). Define this expectation value as $\langle S^\dagger S \rangle = v^2 / 2$. We require $\dim(\mathcal{V}) + 1 \leq \dim(S)$ so that all of the components of the vector become massive in the presence of a scalar field obtaining a vev, through eaten Goldstone components of S .⁹

One can use the global symmetry rotations on S to rotate the new vev to a (uneaten) component of S , denoted s . The interaction of s with $H^\dagger H$ cannot be forbidden by an explicit G breaking without violating our assumptions. This would introduce highly constrained low energy effects

⁹ An additional non goldstone incomplete scalar multiplet is famously required when introducing a vev in this manner [30–32].

into the SMEFT through the vev v' leading to the vector mass matrix. A vacuum misalignment [33] is assumed to make the vector mass matrix is symmetric under G in this work. This results in the Higgs portal coupling not being suppressed by a $G_Q \times G_L$ breaking spurion. Concretely consider

$$\mathcal{L}_{SH} = (D^\mu S)^\dagger (D_\mu S) - \frac{\lambda'}{4} (S^\dagger S - \frac{v'^2}{2})^2 + \lambda_{SH} S^\dagger S H^\dagger H. \quad (23)$$

Here the covariant derivative is $D^\mu = \partial^\mu + ig_V \mathcal{V}_a h^a$ with h_a an abstract group generator that defines the non-trivial representations that the \mathcal{V} multiplet carries. S is expanded as $S = (\cdots, v' + s + \cdots) / \sqrt{2} + h'_a \rho_a$ where ρ_a corresponds to the goldstone components of the S multiplet that are eaten to generate the vector mass, and the \cdots fill out the full dimension of S . The vev v' must be arranged to break the $\dim(\mathcal{V})$ h'_a generators. Simultaneously v' must not break the G subgroup, so $g_a \langle S \rangle = 0$, where the generators of G are denoted g_a .¹⁰ Integrating out s after UV symmetry breaking gives

$$\Delta \mathcal{L}_6 = -\frac{2\lambda_{SH}^2}{\lambda' m_s^2} Q_{H\Box} - \frac{4}{\lambda'} (\mathcal{V}_\mu \mathcal{V}^\mu)^2 + \cdots \quad (24)$$

in addition to the operators induced by integrating out the vector field. Here the scalar mass is $m_s^2 = \lambda' v'^2/2$. In addition, \mathcal{L}_4 terms are induced that require a finite redefinition of λ and v in the SM to rearrange \mathcal{L}_{SM} back into standard form. Note the sizable vector four point interaction, that is enhanced in the $\lambda' \rightarrow 0$ limit, indicating unitarity violation when the UV Higgs is integrated out of the spectrum.

In order to avoid assuming a UV Higgs mechanism, we can consider a composite massive vector generated by a hypothetical UV strong sector, with spin-1/2 constituents Ψ , so that the vector fields are $\mathcal{V}^\mu \sim \langle \bar{\Psi} \gamma^\mu \Psi \rangle$ condensates. This composite field carries at least one non-trivial representation under one of the groups $G_Q, G_L, SU(3)_C$ or $SU(2)_L$ to reduce the SMEFT operator profile to one operator. Denote this non-trivial representation as \mathbf{N} , and the corresponding group as G' . The Ψ are charged under G' or a larger group H with $H \supset G'$.

We can consider G' or the proper subgroup case where $G' \subset H$ without loss of generality with the following arguments. The Ψ belongs to $SU(3)$, and $\mathbf{N} \in \{\mathbf{3}, \bar{\mathbf{3}}, \mathbf{6}, \mathbf{8}\}$, or $SU(2)$ with $\mathbf{N} = \{\mathbf{2}, \mathbf{3}\}$ for the vector fields of interest. The non-trivial representations in \mathbf{N} can be generated from tensor products of the Ψ irreducible representations. In the case where the Ψ belongs to $SU(3)$ we denote the irreducible representations as \mathbf{P}, \mathbf{R} , which need not have the same dimension. When \mathbf{N} is generated by $\mathbf{P} \otimes \bar{\mathbf{P}}$ the singlet representation is also generated in the tensor product. A color singlet sibling under $SU(3)_C$ is expected with

a mass proximate to a color octet vector, which induces a number of operators in \mathcal{L}_6 when integrated out. Similarly, a flavor singlet sibling under a flavor group is also expected for flavor octets. Interestingly, the flavor $\mathbf{8}$ vector fields we considered all have zero $U(1)_Y$ charges, so their flavor singlet siblings with the same $U(1)_Y$ charge are not forbidden by the flavor symmetry to have the coupling with the corresponding quark bi-linear and also with the $J_{H,\mu}$ of vanishing $U(1)_Y$ charge, inducing more than one operator in \mathcal{L}_6 when integrated out. When $\mathbf{N} \in \mathbf{P} \otimes \mathbf{P}$ multiple representations result, for example in the case of $\mathbf{P} = \mathbf{3}$, the $\bar{\mathbf{3}}$ and $\mathbf{6}$ fields are simultaneously present. Such fields ($\mathcal{V}_{X,XI}$) can induce the same operator when integrated out. On the other hand, *these fields necessarily carry $U(1)_Y$, and thus have a cut off scale proximate to the massive vectors mass scale for the cases consistent with our assumptions.*¹¹ Next we consider the cases when the non-trivial representation is generated by bi-linears of Ψ carrying representations of unequal dimension $\mathbf{N} \in \mathbf{P} \otimes \mathbf{R}$. By inspection of the tensor products of $SU(3)$ with triality 0 and 1 [38] it is possible to generate each $\mathbf{N} \in \{\mathbf{3}, \bar{\mathbf{3}}, \mathbf{6}, \mathbf{8}\}$ for $SU(3)$ in such a manner. However, for each \mathbf{P} and \mathbf{R} one can also form a condensate $\langle \bar{\Psi} \gamma_\mu \Psi \rangle$ with zero $U(1)_Y$ charge from the product $\mathbf{P} \otimes \bar{\mathbf{P}}, \mathbf{R} \otimes \bar{\mathbf{R}}$. Two more pure singlet spin one bound states proximate in mass to M_V are expected in the spectrum, unless forbidden by another symmetry.¹² Restricting the discussion for non-trivial $SU(2)$ representations to the vector cases that do not carry $U(1)_Y$ and induce one operator at tree level, we are left with the field \mathcal{V}_{IX}^1 . Further, \mathcal{V}_{IX}^1 has a large flavor breaking spurion proportional to the top Yukawa generating more operators at tree level when integrated out, see Table V.

For all of these reasons, orphaned vector fields with non-trivial representations of the SM symmetry groups demand siblings and a “good UV home”.

B. Spin 1/2 states

If heavy spin-1/2 states are integrated out, the mass¹³ of the massive fermion(s) (denoted M with $M \gg v$) must be introduced in some manner. As discussed in the previous section, a chiral fermion with a UV Higgs mechanism induces more operators at tree level when integrating out the UV scalar field. In this section, we confine the discussion to general vector like fermions. The general Lagrangian associated with a pair of heavy vector-like quark (VLQ) denoted by Q_L, Q_R that are flavor singlets includes

$$\mathcal{L}_Q = \mathcal{L}_Q^0 + \mathcal{L}_Q^{\text{int}}, \quad (25)$$

¹⁰ In general one expects the symmetry breaking pattern to be such that there will be uneaten goldstone bosons, or additional massive vectors in the spectrum. Here we are considering an exceptional minimal spectrum when examining the one operator question.

¹¹ This is a generic expectation if the Ψ are charged under $U(1)_Y$.

¹² For example, some of the expected spectra degeneracy can be lifted in analogy to the η' [34].

¹³ Here we are referring to the dominant component of the mass of the fermion from the new sector. In addition, there will be mass contributions and splitting proportional to $\sim v$. As such effects do not act to reduce the IR SMEFT operator profile, we neglect these contributions.

Case	SU(2) _L	U(1) _Y	J_L^Q	Q_{uH}	Q_{dH}	$Q_{Hq}^{(1)}$	$Q_{Hq}^{(3)}$	
$Q_I^{(1)}$	1	$-\frac{1}{3}$	$\bar{Q}_L H$		✓	✓	✓	
$Q_I^{(3)}$	3	$-\frac{1}{3}$	$\sigma^I \bar{Q}_L H$	✓	✓	✓	✓	
$Q_{II}^{(1)}$	1	$\frac{2}{3}$	$\bar{Q}_L H^*$	✓		✓	✓	
$Q_{II}^{(3)}$	3	$\frac{2}{3}$	$\sigma^I \bar{Q}_L H^*$	✓	✓	✓	✓	
Case	SU(2) _L	U(1) _Y	J_R^Q	Q_{uH}	Q_{dH}	Q_{Hu}	Q_{Hd}	Q_{Hud}
Q_{III}	2	$\frac{1}{6}$	$\bar{u}_R H^T$	✓	✓	✓	✓	✓
Q_{IV}	2	$\frac{1}{6}$	$\bar{d}_R H^\dagger$	✓	✓	✓	✓	✓
Q_V	2	$\frac{7}{6}$	$\bar{u}_R H^\dagger$	✓		✓		
Q_{VI}	2	$-\frac{5}{6}$	$\bar{d}_R H^T$		✓		✓	

TABLE VII. Tree level \mathcal{L}_6 operators induced in the SMEFT with massive quarks integrated out.

Case	SU(2) _L	U(1) _Y	G_Q	J_R^Q	Q_{uH}	Q_{dH}	Q_{Hu}	Q_{Hd}
Q_{VII}	2	$\frac{1}{6}$	(3,1,1)	$\bar{u}_R H^T$	✓		✓	
Q_{VIII}	2	$\frac{1}{6}$	(1,3,1)	$\bar{d}_R H^\dagger$		✓		✓

TABLE VIII. Tree level \mathcal{L}_6 operators induced in the SMEFT with massive quarks integrated out in some sample cases with flavor quantum numbers, see Refs. [35–37] for more discussion on the phenomenology of these fields.

where

$$\mathcal{L}_Q^0 = \bar{Q}_L i \not{D} Q_L + \bar{Q}_R i \not{D} Q_R - M (\bar{Q}_L Q_R + \bar{Q}_R Q_L) \quad (26)$$

for the SU(2)_L singlet and doublets and

$$\mathcal{L}_Q^0 = \text{Tr} [\bar{Q}_L i \not{D} Q_L + \bar{Q}_R i \not{D} Q_R] - M \text{Tr} [\bar{Q}_L Q_R + \bar{Q}_R Q_L] \quad (27)$$

for the **3** of SU(2)_L. The interaction term $\mathcal{L}_Q^{\text{int}}$ for the VLQs to the SM fermions through the Higgs doublet is defined as

$$\mathcal{L}_Q^{\text{int}} = J_L^Q Q_R + J_R^Q Q_L + \text{h.c.} \quad (28)$$

The requirement that the action be stationary under variations of the heavy VLQ fields \bar{Q}_L, \bar{Q}_R results in two coupled EOMs:

$$i \not{D} Q_L - M Q_R + (J_R^Q)^\dagger = 0, \quad (29)$$

$$i \not{D} Q_R - M Q_L + (J_L^Q)^\dagger = 0. \quad (30)$$

Mathematically, the coupled Eqns. (29) and (30) can be solved iteratively. Taking the limit of large M , one can expand the classical solutions schematically as

$$Q_R = \frac{(J_R^Q)^\dagger}{M} + \frac{i \not{D}}{M^2} (J_L^Q)^\dagger + \dots, \quad (31)$$

$$Q_L = \frac{(J_L^Q)^\dagger}{M} + \frac{i \not{D}}{M^2} (J_R^Q)^\dagger + \dots. \quad (32)$$

When substituted back into Eqn. (25) the effect of the leading term in these solutions vanishes due to chirality.

Case	SU(2) _L	U(1) _Y	$J_L^{\mathcal{L}}$	$Q_{Hl}^{(1)}$	$Q_{Hl}^{(3)}$	Q_{eH}	$Q_{He}^{(1)}$
$\mathcal{L}_I^{(1)}$	1	-1	$\bar{L}_L H$	✓	✓	✓	
$\mathcal{L}_I^{(3)}$	3	-1	$\sigma^I \bar{L}_L H$	✓	✓	✓	
Case	SU(2) _L	U(1) _Y	$J_R^{\mathcal{L}}$	$Q_{Hl}^{(1)}$	$Q_{Hl}^{(3)}$	Q_{eH}	$Q_{He}^{(1)}$
\mathcal{L}_{III}	2	$-\frac{1}{2}$	$\bar{e}_R H^\dagger$			✓	✓
\mathcal{L}_{IV}	2	$-\frac{3}{2}$	$\bar{e}_R H^T$			✓	✓

TABLE IX. Tree level \mathcal{L}_6 operators induced in the SMEFT with massive leptons integrated out.

We generically find that multiple operators are induced at tree level when integrating out a vector like fermion. The cases where the vector like quark do not carry flavor quantum numbers are shown in Table VII. In the cases that the VLQs carry flavor quantum numbers, previously discussed in Refs. [35–37], multiple operators are again obtained. We show some sample cases of this type in Table VIII. Multiple operators at tree level are also obtained in the case of integrating out vector like leptons, see Table IX.

C. Spin 0 states

Unlike the cases of massive vectors and spin-1/2 fields, a massive scalar can couple into the SM through a number of interactions and naively generate many operators in the IR SMEFT matching limit. However, the examples (\mathcal{S}_A , \mathcal{S}_B and \mathcal{S}_C in Table X) discussed in Refs. [8, 39] show that only one operator Q_H , can be obtained if an explicit scale is introduced without a dynamical origin to give the scalar a mass. For instance, \mathcal{S}_A couples through linear and bilinear interactions in the full multi-scalar potential, denoted $V(H, \mathcal{S}_A)$ in Table X. To reduce the operator profile of \mathcal{S}_A to one operator, it is assumed that \mathcal{S}_A has a discrete or additional U(1) symmetry. Such a symmetry forbids a large number of four-fermion operators at tree level, and also a number of linear \mathcal{S} interactions in the scalar potential that otherwise generate $Q_{H\Box}$. Similarly, $\mathcal{S}_{B,C}$ also have minimal one operator profiles containing only Q_H . However, this again follows from the UV scale being introduced without a dynamical origin. In all these cases, a hierarchy problem in the UV sector is also introduced.

Table X also lists the cases of flavor singlet scalar fields that couple to through the $\mathcal{S}^2 H^\dagger H$ interaction and in addition have an independent $\mathcal{S} H^\dagger H$ interaction via a dimensionfull coupling. In these cases, the operators Q_H and $Q_{H\Box}$ are simultaneously produced in tree level matchings.

As in the case of massive vectors and fermions, scalars can carry non-trivial representations under G_Q or G_L to isolate the coupling to a single fermion bi-linear. These states have been studied previously in Refs. [40–44]. To avoid an explicit breaking of G_Q or G_L in this coupling, all of these states carry at least two non-trivial representations under the flavor (G_Q or G_L) or gauge (SU(3)_C or SU(2)_L) groups. For instance, consider integrating

Case	SU(2) _L	U(1) _Y	Couplings	Q_H	$Q_{H\Box}$
\mathcal{S}_A	2	1/2	$V(H, \mathcal{S}_A)$	✓	
\mathcal{S}_B	4	3/2	$(H^3)^\dagger \mathcal{S}_B + \text{h.c.}$	✓	
\mathcal{S}_C	4	1/2	$H^\dagger \mathcal{S}_C H^\dagger H + \text{h.c.}$	✓	
\mathcal{S}_I^1	1	0	$(\Lambda_S \mathcal{S}_I + (\mathcal{S}_I)^\dagger \mathcal{S}_I) H^\dagger H$	✓	✓
\mathcal{S}_I^3	3	0	$\Lambda_S \mathcal{S}_I \sigma H^\dagger H, (\mathcal{S}_I)^\dagger \mathcal{S}_I H^\dagger H$	✓	✓
\mathcal{S}_{II}^1	1	-1	$\Lambda_S \mathcal{S}_{II} H^T H, (\mathcal{S}_{II})^\dagger \mathcal{S}_{II} H^\dagger H$	✓	✓
\mathcal{S}_{II}^3	3	-1	$\Lambda_S \mathcal{S}_{II} \sigma H^T H, (\mathcal{S}_{II})^\dagger \mathcal{S}_{II} H^\dagger H$	✓	✓

TABLE X. \mathcal{L}_6 operators obtained at tree level when flavor and colour singlet scalars are integrated out. Λ_S indicates a dimensionfull coupling.

Case	SU(3) _C	SU(2) _L	U(1) _Y	G_Q	Couples to	Op
\mathcal{S}_{III}	3	1	-4/3	(3,1,1)	$u_R u_R$	Q_{uu}
\mathcal{S}_{IV}	$\bar{6}$	1	-4/3	($\bar{6}$,1,1)	$u_R u_R$	Q_{uu}
\mathcal{S}_V	3	1	2/3	(1,3,1)	$d_R d_R$	Q_{dd}
\mathcal{S}_{VI}	$\bar{6}$	1	2/3	(1, $\bar{6}$,1)	$d_R d_R$	Q_{dd}
Case	SU(3) _C	SU(2) _L	U(1) _Y	G_L	Couples to	Op
\mathcal{S}_{VII}	1	1	2	(1, $\bar{6}$)	$e_R e_R$	Q_{ee}

TABLE XI. The cases where a single \mathcal{L}_6 operator is generated at tree level for different scalar representations that are not singlets under the flavor group, without the insertion of spurion Yukawa fields, from Ref. [42].

out “di-quark” states of this form discussed in Ref. [42] at tree level. A scalar current operator of the form $\bar{\psi}_{1L} \psi_{2R} \bar{\psi}_{2R} \psi_{1L}$ is directly obtained. This operator can be projected into the Warsaw basis via Fierz transformation,

$$(\bar{\psi}_{1L} \psi_{4R})(\bar{\psi}_{3R} \psi_{2L}) = -\frac{1}{2}(\bar{\psi}_{1L} \gamma_\mu \psi_{2L})(\bar{\psi}_{3R} \gamma^\mu \psi_{4R}). \quad (33)$$

As the “di-quark” scalars are in non-trivial representations under SU(2)_L and/or SU(3)_C groups, the index associated with these symmetries are not contracted between the fermions in each vector current, c.f. the right hand side of Eqn. (33). When reducing to the Warsaw basis one uses the SU(3) and SU(2) relations

$$T_{ij}^A T_{kl}^A = \frac{1}{2} \delta_{il} \delta_{jk} - \frac{1}{6} \delta_{ij} \delta_{kl}, \quad (34)$$

$$\sigma_{jk}^I \sigma_{mn}^I = 2 \delta_{jn} \delta_{mk} - \delta_{jk} \delta_{mn}. \quad (35)$$

Concretely, performing this mapping for the “di-quark” scalars that couple to $\bar{u}_R Q_L$ and $\bar{d}_R Q_L$ induce the operators $Q_{qu}^{(1,8)}$ and $Q_{qd}^{(1,8)}$ respectively. Similarly, the “di-quark” scalars coupling to $Q_L Q_L$ generate $Q_{qq}^{(1,3)}$. On the other hand, exceptional cases that can generate only one operator do exist in “di-quark” scalars that couple to right handed SU(2)_L bi-linears of the same fermion field i.e. to pairs of u_R , d_R and e_R . These scalars can induce the single operator Q_{uu} , Q_{ee} , Q_{dd} that are defined in the Warsaw basis, see the examples in Table XI.

In spite of only Q_H being induced at tree level in cases $\mathcal{S}_{A,B,C}$ and only one of the operators Q_{uu} , Q_{dd} and Q_{ee}

obtained at tree level in the cases $\mathcal{S}_{III} - \mathcal{S}_{VII}$, the arguments based on the mass scale generation from a UV Higgs mechanism with an associated extra scalar degree of freedom still hold. The heavy scalar (\mathcal{S}) can be embedded in a larger scalar multiplet \mathcal{S}' that develops a vev, or not so embedded, when a UV Higgs mechanism is invoked to introduce a new scale $\Lambda \gg v$. Due to the fact that any field obtaining a vev with its self conjugate forms a singlet under G this leads to $Q_{H\Box}$ (as shown in Eqn. (24)) in either case, in addition to any matchings of \mathcal{S} integrated out at tree level.

Alternatively, if a strong sector is present and the “di-quark” scalar is composite, then the arguments in favor of “sibling” fields imply an extended spectrum that generically contains singlet composite states. Additionally, in the presence of a confining strong sector, both spin-0 and spin-1 composite states are expected to be embedded in a spin tower [46]. In addition, some form of dimensional transmutation can be used to generate a scale. This can take place in the context of weaker couplings using the Coleman-Weinberg (CW) mechanism [45], or in the case with stronger couplings with a mechanism similar to the generation of Λ_{QCD} . Of course, the (weak coupling version) of CW requires multiple couplings and generically a non-minimal UV particle spectrum.

It is also important to notice that there are scalar four-fermion current operators with the chiral structure $(\bar{L}R)(\bar{L}R)$ and $(\bar{L}R)(\bar{R}L)$ defined in the Warsaw basis. However, these operators are not constructed out of a pair of bi-linears with the same SM field content. As a result, additional vector current operators are induced when the $(\bar{L}R)(\bar{L}R)$ and $(\bar{L}R)(\bar{R}L)$ operators are obtained with a tree level exchange. Nevertheless, the presence of multiple four-fermion operators induced at tree level from the projection of some four-fermion scalar currents into the form of the Warsaw basis is clearly a more basis dependent conclusion than other arguments made in this paper.

IV. CONCLUSIONS

Driven by the question: “Can one obtain only one dimension six operator in the SMEFT from a consistent tree level matching onto an unknown new physics sector?”, in this paper we have examined the non-minimal character of the SMEFT.

We addressed this question using a (G and CP) symmetry assumption to accommodate the large set of lower energy measurements that probe the symmetry breaking pattern of the SM into the TeV mass scale range and beyond. We have focused on the tree level matchings capturing the consistent IR limit of a new physics sector. Due to the extensive mixing of the operators in \mathcal{L}_6 under renormalization [3–5], the SMEFT clearly has a non-minimal character once loop induced effects are considered, requiring many operators for consistent lower energy data analysis.

We have uncovered some cases where only one operator

(neglecting flavor indices) is naively induced at tree level. These operators in the Warsaw basis are $(H^\dagger H)^3$, or of the four-fermion form, and come about due to a massive scalar or vector field having non-trivial representations under the symmetry groups of the SM. We have found that vector fields can carry non-zero $U(1)_Y$ charge to reduce the operator profile by avoiding Higgs scalar currents being induced, but these fields have severe unitarity problems due to the lack of a three-point vector self-interaction. This indicates the presence of large non-perturbative matching corrections in addition to tree level matching effects. On the other hand, when the massive vector fields are not charged under $U(1)_Y$, flavor symmetries can be introduced to reduce the IR SMEFT operator profile. In this case, a spurion symmetry breaking analysis shows scalar currents are still induced, leading to more operators at tree level. In practice, fitting to one pure lepton or down quark four-fermion operator is not as poorly motivated as fitting to up quark four-fermion operators due to the relative magnitudes of the flavor breaking spurions in each case.

In contrast to the vector fields, the presence of scalar fields in a UV sector do not directly cause severe unitarity problems. Integrating them out could have a relatively minimal operator profile, i.e. only Q_{uu} , Q_{dd} or Q_{ee} is induced at tree level in the cases shown in Table XI, and only Q_H in some cases in Table X. However, requiring a mass generation mechanism for these fields would lead to more matching contributions to the SMEFT operators. The scenario of a UV Higgs mechanism, if present, generically induces more operators that are constructed out of the SM Higgs field at tree level. This occurs without a suppression by a $G_Q \times G_L$ symmetry breaking spurion. When a UV Higgs mechanism is avoided by assuming compositeness and a new strong interaction, we have argued that the requirement of non-trivial representations for the vector and scalar fields to reduce the operator profile would indicate the presence of an extended spectrum of the composite states - including singlet fields - that couple through many SM portals. This would lead to more operators with tree level matchings when the extended spectrum is integrated out.

The SMEFT is a complicated field theory. It is natural and reasonable to seek a reduction of this complexity to use in data analyses in the SMEFT framework. Using symmetry assumptions is widely accepted. We have examined in this work if an alternative ad-hoc approach of using “one operator at a time” in data analyses can be representative of a consistent tree level matching to an unknown new physics sector. Our results show that the SMEFT has a non-minimal character quite generically and thus this approach should be avoided, if possible. To ensure the right conclusions are being drawn on the degree of constraint on unknown UV physics sectors, multiple operators should be retained in data analyses. Fortunately global data analyses in the SMEFT can already be performed with multiple operators, by using symmetries to simplify the number of parameters present. Further the growing LHC data set makes such global analyses even more feasible to execute

in practice. In some cases, the resulting constraints can be relaxed by orders of magnitude [47, 48] compared to a “one operator at a time” analysis. Nevertheless, our analysis shows that retaining multiple operators is preferred, and a relaxation of constraints can be required to obtain a consistent IR limit of an underlying UV physics sector, when a dynamical origin of the UV scales introduced is demanded.

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- [1] W. Buchmuller and D. Wyler, Nucl. Phys. B **268**, 621 (1986). doi:10.1016/0550-3213(86)90262-2
 - [2] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, JHEP **1010**, 085 (2010) doi:10.1007/JHEP10(2010)085 [arXiv:1008.4884 [hep-ph]].
 - [3] E. E. Jenkins, A. V. Manohar and M. Trott, JHEP **1310**, 087 (2013) doi:10.1007/JHEP10(2013)087 [arXiv:1308.2627 [hep-ph]].
 - [4] E. E. Jenkins, A. V. Manohar and M. Trott, JHEP **1401**, 035 (2014) doi:10.1007/JHEP01(2014)035 [arXiv:1310.4838 [hep-ph]].
 - [5] R. Alonso, E. E. Jenkins, A. V. Manohar and M. Trott, JHEP **1404**, 159 (2014) doi:10.1007/JHEP04(2014)159 [arXiv:1312.2014 [hep-ph]].
 - [6] L. Lehman, Phys. Rev. D **90**, no. 12, 125023 (2014) doi:10.1103/PhysRevD.90.125023 [arXiv:1410.4193 [hep-ph]].
 - [7] L. Lehman and A. Martin, JHEP **1602**, 081 (2016) doi:10.1007/JHEP02(2016)081 [arXiv:1510.00372 [hep-ph]].
 - [8] B. Henning, X. Lu and H. Murayama, JHEP **1601**, 023 (2016) doi:10.1007/JHEP01(2016)023 [arXiv:1412.1837 [hep-ph]].
 - [9] B. Henning, X. Lu, T. Melia and H. Murayama, doi:10.1007/s00220-015-2518-2 arXiv:1507.07240 [hep-th].
 - [10] B. Henning, X. Lu, T. Melia and H. Murayama, arXiv:1512.03433 [hep-ph].
 - [11] G. F. Giudice, B. Gripaios and R. Sundrum, JHEP **1108**, 055 (2011) doi:10.1007/JHEP08(2011)055 [arXiv:1105.3161 [hep-ph]].
 - [12] R. S. Chivukula and H. Georgi, Phys. Lett. B **188**, 99 (1987). doi:10.1016/0370-2693(87)90713-1
 - [13] G. D’Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B **645**, 155 (2002) doi:10.1016/S0550-3213(02)00836-2 [hep-ph/0207036].
 - [14] V. Cirigliano, B. Grinstein, G. Isidori and M. B. Wise, Nucl. Phys. B **728**, 121 (2005) doi:10.1016/j.nuclphysb.2005.08.037 [hep-ph/0507001].
 - [15] C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985). doi:10.1103/PhysRevLett.55.1039
 - [16] C. Jarlskog, Z. Phys. C **29**, 491 (1985). doi:10.1007/BF01565198
 - [17] S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979). doi:10.1103/PhysRevLett.43.1566
 - [18] F. Wilczek and A. Zee, Phys. Rev. Lett. **43**, 1571 (1979). doi:10.1103/PhysRevLett.43.1571
 - [19] B. M. Gavela, E. E. Jenkins, A. V. Manohar and L. Merlo, Eur. Phys. J. C **76**, no. 9, 485 (2016)

- doi:10.1140/epjc/s10052-016-4332-1 [arXiv:1601.07551 [hep-ph]].
- [20] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974). doi:10.1103/PhysRevLett.32.438
 - [21] B. Grinstein, A. L. Kagan, M. Trott and J. Zupan, Phys. Rev. Lett. **107**, 012002 (2011) doi:10.1103/PhysRevLett.107.012002 [arXiv:1102.3374 [hep-ph]].
 - [22] B. Grinstein, A. L. Kagan, J. Zupan and M. Trott, JHEP **1110**, 072 (2011) doi:10.1007/JHEP10(2011)072 [arXiv:1108.4027 [hep-ph]].
 - [23] F. del Aguila, J. de Blas and M. Perez-Victoria, JHEP **1009**, 033 (2010) doi:10.1007/JHEP09(2010)033 [arXiv:1005.3998 [hep-ph]].
 - [24] E. G. Floratos, D. A. Ross and C. T. Sachrajda, Nucl. Phys. B **129**, 66 (1977) Erratum: [Nucl. Phys. B **139**, 545 (1978)]. doi:10.1016/0550-3213(78)90367-X, 10.1016/0550-3213(77)90020-7
 - [25] L. D. Landau, Niels Bohr and the Development of Physics, edited by W. Pauli (Pergamon Press, London, 1955).
 - [26] K. G. Wilson, Rev. Mod. Phys. **47**, 773 (1975). doi:10.1103/RevModPhys.47.773
 - [27] B. W. Lee, C. Quigg and H. B. Thacker, Phys. Rev. D **16**, 1519 (1977). doi:10.1103/PhysRevD.16.1519
 - [28] A. Schuessler and D. Zeppenfeld, In *Karlsruhe 2007, SUSY 2007* 236-239 [arXiv:0710.5175 [hep-ph]].
 - [29] S. El Hedri, W. Shepherd and D. G. E. Walker, arXiv:1412.5660 [hep-ph].
 - [30] P. W. Higgs, Phys. Rev. Lett. **13**, 508 (1964). doi:10.1103/PhysRevLett.13.508
 - [31] F. Englert and R. Brout, Phys. Rev. Lett. **13**, 321 (1964). doi:10.1103/PhysRevLett.13.321
 - [32] G. S. Guralnik, C. R. Hagen and T. W. B. Kibble, Phys. Rev. Lett. **13**, 585 (1964). doi:10.1103/PhysRevLett.13.585
 - [33] D. B. Kaplan and H. Georgi, Phys. Lett. **136B**, 183 (1984). doi:10.1016/0370-2693(84)91177-8
 - [34] G. 't Hooft, Phys. Rev. Lett. **37**, 8 (1976). doi:10.1103/PhysRevLett.37.8
 - [35] Y. Grossman, Y. Nir, J. Thaler, T. Volansky and J. Zupan, Phys. Rev. D **76**, 096006 (2007) doi:10.1103/PhysRevD.76.096006 [arXiv:0706.1845 [hep-ph]].
 - [36] E. Gross, D. Grossman, Y. Nir and O. Vitells, Phys. Rev. D **81**, 055013 (2010) doi:10.1103/PhysRevD.81.055013 [arXiv:1001.2883 [hep-ph]].
 - [37] J. M. Arnold, B. Fornal and M. Trott, JHEP **1008**, 059 (2010) doi:10.1007/JHEP08(2010)059 [arXiv:1005.2185 [hep-ph]].
 - [38] R. Slansky, Phys. Rept. **79**, 1 (1981). doi:10.1016/0370-1573(81)90092-2
 - [39] M. Gorbahn, J. M. No and V. Sanz, JHEP **1510**, 036 (2015) doi:10.1007/JHEP10(2015)036 [arXiv:1502.07352 [hep-ph]].
 - [40] W. Buchmuller, R. Ruckl and D. Wyler, Phys. Lett. B **191**, 442 (1987) Erratum: [Phys. Lett. B **448**, 320 (1999)]. doi:10.1016/S0370-2693(99)00014-3, 10.1016/0370-2693(87)90637-X
 - [41] A. J. Davies and X. G. He, Phys. Rev. D **43**, 225 (1991). doi:10.1103/PhysRevD.43.225
 - [42] J. M. Arnold, M. Pospelov, M. Trott and M. B. Wise, JHEP **1001**, 073 (2010) doi:10.1007/JHEP01(2010)073 [arXiv:0911.2225 [hep-ph]].
 - [43] J. M. Arnold, B. Fornal and M. B. Wise, Phys. Rev. D **88**, 035009 (2013) doi:10.1103/PhysRevD.88.035009 [arXiv:1304.6119 [hep-ph]].
 - [44] J. M. Arnold, B. Fornal and M. B. Wise, Phys. Rev. D **87**, 075004 (2013) doi:10.1103/PhysRevD.87.075004 [arXiv:1212.4556 [hep-ph]].
 - [45] S. R. Coleman and E. J. Weinberg, Phys. Rev. D **7**, 1888 (1973). doi:10.1103/PhysRevD.7.1888
 - [46] M. B. Einhorn, Phys. Rev. D **14**, 3451 (1976). doi:10.1103/PhysRevD.14.3451
 - [47] Z. Han and W. Skiba, Phys. Rev. D **71**, 075009 (2005) doi:10.1103/PhysRevD.71.075009 [hep-ph/0412166].
 - [48] L. Berthier, M. Bjørn and M. Trott, JHEP **1609**, 157 (2016) doi:10.1007/JHEP09(2016)157 [arXiv:1606.06693 [hep-ph]].